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Relativistic description of the double charmonium production in e^+e^- annihilation

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ABSTRACT

New evaluation of the relativistic effects in the double production of S-wave charmonium states is performed on the basis of perturbative QCD and the relativistic quark model. The main improvement consists in the exact account of properties of the relativistic meson wave functions. For the gluon and quark propagators entering the production vertex function we use a truncated expansion in the ratio of the relative quark momenta to the center-of-mass energy $\sqrt{s}$ up to the second order. The exact relativistic treatment of the wave functions makes all such second order terms convergent, thus allowing the reliable calculation of their contributions to the production cross section. Compared to the nonrelativistic calculation we obtain a significant increase of the cross sections for the S-wave double charmonium production. This brings new theoretical results in good agreement with the available experimental data.

The production processes of mesons and baryons containing heavy b and c quarks in different reactions are under intensive study at present [1–4]. The experimental investigation of the double charmonium production in e^+e^- annihilation by BaBar and Belle Collaborations revealed a discrepancy between the measured cross sections and theoretical results obtained in the nonrelativistic approximation in QCD [5–7]. Various efforts have been undertaken to improve the theoretical calculations. They include the evaluation of radiative corrections of order α_s and the investigation of relativistic effects connected with the relative motion of the heavy quarks forming the vector and pseudoscalar quarkonia [7–19]. As a result, the difference between theory and experiment for the value of the center-of-mass energy $\sqrt{s} = 10.6$ GeV was essentially decreased [7,12,13,15]. Moreover, the new theoretical analysis carried out in Refs. [19,20] shows that the inclusion of order α_s and relativistic corrections decreases the discrepancy between theory and experiment at the present level of precision. But despite this fact there exists the frequently debated question connected with the calculation of the relativistic corrections in the production cross section. It is related to the determination of the specific parameter $\langle \mathbf{p}^2\rangle = \int \mathbf{p}^2 \Psi_0^{\mathcal{P},\mathcal{V}}(\mathbf{p})\, d\mathbf{p}/(2\pi)^3$ emerging after the expansion of all quantities in the production amplitude in the relative quark momenta $\mathbf{p}$ and $\mathbf{q}$ [15,20–22], where $\Psi_0^{\mathcal{V},\mathcal{P}}$ are the vector and pseudoscalar charmonium wave functions in the rest frame. The divergence of this integral required the use of a regularization procedure (dimensional regularization is commonly used) which led to a definite uncertainty of the evaluation. Moreover, the large value of the relativistic contribution obtained in the previous studies [7,15] evidently rises a question about the convergence of the expansion in the heavy quark velocity. In this Letter we propose an alternative approach to the calculation of relativistic effects based on the relativistic quark model [23–27] and perturbative QCD. It uses a truncated expansion in relative momenta $\mathbf{p}$ and $\mathbf{q}$ and thus avoids divergent integrals in the relativistic contribution of the second order.

Define the four momenta of the produced $c, \bar{c}$ quarks forming the vector and pseudoscalar charmonia in terms of total momenta $P(Q)$ and relative momenta $p(q)$ as follows:

$$p_{1,2} = \frac{1}{2}P \pm p, \quad (p \cdot P) = 0, \qquad q_{1,2} = \frac{1}{2}Q \pm q, \quad (q \cdot Q) = 0, \tag{1}$$

where $p = L_P(0, \mathbf{p})$, $q = L_P(0, \mathbf{q})$ are the four-momenta obtained from the rest frame four-momenta $(0, \mathbf{p})$ and $(0, \mathbf{q})$ by the Lorentz transformation to the system moving with the momenta P, Q. Then the production amplitude of the S-wave vector and pseudoscalar

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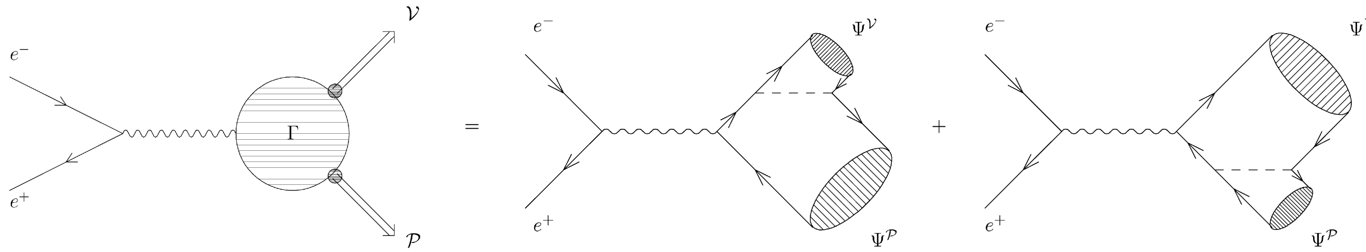

Fig. 1. The production amplitude of a pair of charmonium states ($\mathcal{V}$ denotes the vector meson and $\mathcal{P}$ the pseudoscalar meson) in e^+e^- annihilation. The wave line shows the virtual photon and the dashed line corresponds to the gluon. Γ is the production vertex function.

charmonium states, shown in Fig. 1, can be presented in the form [15,28,29]:

$$\mathcal{M}(p_-,p_+,P,Q)=\frac{8\pi^2\alpha\alpha_s\mathcal{Q}_c}{3s}\bar v(p_+)\gamma^\beta u(p_-)\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}Sp\{\Psi^{\mathcal{V}}(p,P)\Gamma^\nu(p,q,P,Q)\Psi^{\mathcal{P}}(q,Q)\gamma_\nu\},\tag{2}$$

where α_s is the QCD coupling constant, α is the fine structure constant, $\mathcal{Q}_c$ is the c quark electric charge. The relativistic wave functions of the bound quarks $\Psi^{\mathcal{V},\mathcal{P}}$ accounting for the transformation from the rest frame to the moving one with four momenta P,Q are

$$\Psi^{\mathcal{V}}(p,P)=\frac{\Psi_0^{\mathcal{V}}(\mathbf{p})}{\left[\frac{\epsilon(p)}{m}\frac{(\epsilon(p)+m)}{2m}\right]}\left[\frac{\hat v_1-1}{2}+\hat v_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}-\frac{\hat p}{2m}\right]\hat{\tilde\epsilon}^*(1+\hat v_1)\left[\frac{\hat v_1+1}{2}+\hat v_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}+\frac{\hat p}{2m}\right],\tag{3}$$

$$\Psi^{\mathcal{P}}(q,Q)=\frac{\Psi_0^{\mathcal{P}}(\mathbf{q})}{\left[\frac{\epsilon(q)}{m}\frac{(\epsilon(q)+m)}{2m}\right]}\left[\frac{\hat v_2-1}{2}+\hat v_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}+\frac{\hat q}{2m}\right]\gamma_5(1+\hat v_2)\left[\frac{\hat v_2+1}{2}+\hat v_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}-\frac{\hat q}{2m}\right],\tag{4}$$

where $v_1=P/M_{\mathcal{V}}$, $v_2=Q/M_{\mathcal{P}}$; $\tilde\epsilon$ is the polarization vector of the vector charmonium; $\epsilon(p)=\sqrt{p^2+m^2}$ and m is the c quark mass. The vertex function $\Gamma^\nu(p,P;q,Q)$ at leading order in α_s can be written as a sum of four contributions:

$$\begin{aligned}\Gamma^\nu(p,P;q,Q)&=\gamma_\mu\frac{(\hat r-\hat q_1+m)}{(r-q_1)^2-m^2+i\epsilon}\gamma_\beta D^{\mu\nu}(k_2)+\gamma_\beta\frac{(\hat p_1-\hat r+m)}{(r-p_1)^2-m^2+i\epsilon}\gamma_\mu D^{\mu\nu}(k_2)\\&\quad+\gamma_\beta\frac{(\hat q_2-\hat r+m)}{(r-q_2)^2-m^2+i\epsilon}\gamma_\mu D^{\mu\nu}(k_1)+\gamma_\mu\frac{(\hat r-\hat p_2+m)}{(r-p_2)^2-m^2+i\epsilon}\gamma_\beta D^{\mu\nu}(k_1),\end{aligned}\tag{5}$$

where the gluon momenta are $k_1=p_1+q_1$, $k_2=p_2+q_2$ and $r^2=s=(P+Q)^2=(p_-+p_+)^2$, p_-, p_+ are four momenta of the electron and positron. The dependence on the relative momenta of c-quarks is present both in the gluon propagator $D_{\mu\nu}(k)$ and quark propagators as well as in the relativistic wave functions. One of the main technical difficulties in calculating the production amplitude (2) consists in performing angular integrations, since both gluon and quark propagators in the vertex function (5) contain angles in the denominators. Therefore we expand these propagators in the relative momenta. Such expansion leads to the vertex function containing angles only in numerators and, thus, the angular integrations can be easily performed.

The inverse denominators of quark propagators expanded in the ratio of the relative quark momenta p,q to the energy $\sqrt{s}$ up to the second order can be expressed as follows:

$$\frac{1}{(r-q_{1,2})^2-m^2}=\frac{1}{Z_1}\left[1-\frac{q^2}{Z_1}\pm\frac{2(rq)}{Z_1}+\frac{4(rq)^2}{Z_1^2}+\cdots\right],\tag{6}$$

$$\frac{1}{(r-p_{1,2})^2-m^2}=\frac{1}{Z_2}\left[1-\frac{p^2}{Z_2}\pm\frac{2(rp)}{Z_2}+\frac{4(rp)^2}{Z_2^2}+\cdots\right],\tag{7}$$

where the factors Z_1 and Z_2 differ only due to the bound state corrections:

$$Z_1=\frac{2s+2M_{\mathcal{V}}^2-M_{\mathcal{P}}^2-4m^2}{4},\qquad Z_2=\frac{2s+2M_{\mathcal{P}}^2-M_{\mathcal{V}}^2-4m^2}{4}.\tag{8}$$

Corresponding expansions of the gluon propagators in Eq. (5) with the account of terms of order $O(p^2/s,q^2/s)$ are ($Z=s/4$):

$$\frac{1}{k_{2,1}^2}=\frac{1}{Z}\left[1-\frac{p^2+q^2+2pq}{Z}\pm\frac{(rp)+(rq)}{Z}+\frac{(rp)^2+(rq)^2+2(rp)(rq)}{Z^2}+\cdots\right].\tag{9}$$

We expanded the gluon and quark propagators in the ratio of the relative quark momenta to the center-of-mass energy $\sqrt{s}$ up to the second order terms in the production vertex function (5) but preserved all relativistic factors entering the denominators of the relativistic wave functions (3), (4). This provides the convergence of the resulting momentum integrals. Then keeping the terms of second and fourth order in both variables p and q in the numerator of Eq. (2) from the relativistic wave functions (3)–(4) and second order from the expansions of the quark and gluon propagators, we perform the angular averaging taking into account Eq. (1) and using the following relation:

$$\int p_\mu p_\nu\,d\Omega_{\mathbf{p}}=-\frac{1}{3}\mathbf{p}^2\left(g_{\mu\nu}-\frac{P_\mu P_\nu}{M^2}\right).\tag{10}$$

Then we can write the total production amplitude $\mathcal{M}$ in the form:

$$\mathcal{M}\big(e^+e^- \to \mathcal{P}+\mathcal{V}\big) = \frac{256}{9}\pi^2\alpha\alpha_s\mathcal{Q}_c\frac{\sqrt{4M_{\mathcal{P}}M_{\mathcal{V}}}}{s^2u^2(1-u)^2(M_{\mathcal{V}}+M_{\mathcal{P}})}\bar v(p_+)\gamma^\beta u(p_-)\epsilon_{\sigma\rho\lambda\beta}v_1^\sigma v_2^\rho\tilde\epsilon^{*\lambda}$$
$$\times\int\frac{d\mathbf{p}}{(2\pi)^3}\left(\frac{\epsilon(p)+m}{2\epsilon(p)}\right)\Psi_0^{\mathcal{V}}(\mathbf{p})\int\frac{d\mathbf{q}}{(2\pi)^3}\left(\frac{\epsilon(q)+m}{2\epsilon(q)}\right)\Psi_0^{\mathcal{P}}(\mathbf{q})\left[\frac{T_{13}}{Z_1}+\frac{T_{24}}{Z_2}\right], \tag{11}$$

where T_{13} originates from the sum of the first and third terms in the vertex function (5) and T_{24} from the sum of the second and fourth terms. First, using the Form package [30] we presented T_{13} and T_{24} as a series over the factors Z_1, Z_2, $u = M_{\mathcal{P}}/(M_{\mathcal{P}}+M_{\mathcal{V}})$, $\kappa = m/(M_{\mathcal{P}}+M_{\mathcal{V}})$, $c(p) = [2m/(\epsilon(p)+m)-1] \equiv -\mathbf{p}^2/(\epsilon(p)+m)^2$, $c(q) = [2m/(\epsilon(q)+m)-1] \equiv -\mathbf{q}^2/(\epsilon(q)+m)^2$. The resulting expressions are cumbersome so we omit them here.[1] Then we performed their simplification by neglecting the bound state corrections in the denominators Z_1 and Z_2 (8). This can be done because the value of $\sqrt{s}$ at which the experimental data were obtained is essentially larger than the quark bound state energy. In this approximation which does not influence the accuracy of the calculation (the corresponding error in the cross section at the energy $\sqrt{s} = 10$–11 GeV amounts 0.5%) we have $Z_1 \approx Z_2 \approx s/2$. After such approximation the total cross section for the exclusive production of pseudoscalar and vector charmonium states in e^+e^- annihilation is given by the following analytical expression:

$$\sigma(s) = \frac{8192\pi^3\alpha^2\alpha_s^2\mathcal{Q}_c^2}{2187s^4u^5(1-u)^5}\left\{\left[1-\frac{(M_{\mathcal{V}}+M_{\mathcal{P}})^2}{s}\right]\left[1-\frac{(M_{\mathcal{V}}-M_{\mathcal{P}})^2}{s}\right]\right\}^{3/2}$$
$$\times\left[\int\frac{d\mathbf{p}}{(2\pi)^3}\left(\frac{\epsilon(p)+m}{2\epsilon(p)}\right)\Psi_0^{\mathcal{V}}(\mathbf{p})\int\frac{d\mathbf{q}}{(2\pi)^3}\left(\frac{\epsilon(q)+m}{2\epsilon(q)}\right)\Psi_0^{\mathcal{P}}(\mathbf{q})T(\mathbf{p},\mathbf{q})\right]^2, \tag{12}$$

where the function $T(\mathbf{p},\mathbf{q})$ can be written as follows:

$$T(\mathbf{p},\mathbf{q}) = \sum_{k,l=0}^{2}\omega_{kl}c^k(p)c^l(q) + \frac{(M_{\mathcal{V}}+M_{\mathcal{P}})^2}{s}\sum_{k,l=0}^{2}\rho_{kl}c^k(p)c^l(q)$$
$$+\frac{(M_{\mathcal{V}}+M_{\mathcal{P}})^4}{s^2}\sum_{k,l=0}^{2}\sigma_{kl}c^k(p)c^l(q) + \frac{(M_{\mathcal{V}}+M_{\mathcal{P}})^6}{s^3}\gamma_1c(p)c(q) + \frac{(M_{\mathcal{V}}+M_{\mathcal{P}})^8}{s^4}\gamma_2c(p)c(q). \tag{13}$$

The nonzero values of the coefficients ω_{kl}, ρ_{kl}, σ_{kl}, $\gamma_{1,2}$ are given explicitly in Appendix A.

The momentum integrals entering Eq. (12) are convergent and we calculate them numerically, using the wave functions obtained by the numerical solution of the relativistic quasipotential wave equation [24–26,31]. The exact form of the wave functions $\Psi^{\mathcal{V}}(\mathbf{p})$ and $\Psi^{\mathcal{P}}(\mathbf{q})$ is extremely important for getting the reliable numerical results. It is sufficient to note that the charmonium production cross section $\sigma(s)$ in the nonrelativistic approximation contains the factor $|\Psi^{\mathcal{V}}_{\mathrm{NR}}(0)|^2|\Psi^{\mathcal{P}}_{\mathrm{NR}}(0)|^2$. So, small changes of the numerical values $\Psi^{\mathcal{V}}_{\mathrm{NR}}(0)$ and $\Psi^{\mathcal{P}}_{\mathrm{NR}}(0)$ considerably influence the final result. In the approach based on nonrelativistic QCD this problem is closely related to the determination of the color-singlet matrix elements for the charmonium. Therefore for our calculations we use the charmonium wave functions $\Psi^{\mathcal{V},P}$ obtained with the complete nonperturbative treatment of relativistic effects. For this purpose we consider the quark–antiquark interaction operator constructed in the relativistic quark model in Refs. [24–26]. Thus, in the present study of the production amplitude (2) we keep the relativistic corrections of two types. The first type is determined by several functions depending on the relative quark momenta $\mathbf{p}$ and $\mathbf{q}$ arising from the gluon propagator, the quark propagator and the relativistic meson wave functions. The second type of corrections originate from the nonperturbative treatment of the hyperfine interaction in the quark–antiquark potential which leads to the different wave functions $\Psi_0^{\mathcal{V}}(\mathbf{p})$ and $\Psi_0^{\mathcal{P}}(\mathbf{q})$ for the vector and pseudoscalar charmonium states, respectively. In addition, we systematically accounted the bound state corrections working with the observed masses of the vector and pseudoscalar mesons. The calculated masses of vector and pseudoscalar charmonium states agree well with experimental values [25,32]. Note that all parameters of the model are kept fixed from the previous calculations of the meson mass spectra and decay widths [24,25,27]. The masses of the S-wave charmonium states are: $m_{J/\Psi} = 3.097$ GeV, $m_{\eta_c} = 2.980$ GeV, $m_{\Psi'} = 3.686$ GeV, $m_{\eta_c'} = 3.637$ GeV. The strong coupling constant entering the production amplitude (2) is taken to be $\alpha_s = 0.21$ (see also [7,12]).

Numerical results and their comparison with several previous calculations and experimental data are presented in Table 1. In Refs. [17,19,20] the cross section $\sigma[e^+ + e^- \to J/\Psi + \eta_c]$ was calculated with the values 20.04 fb, 17.5 ± 5.7 fb and $17.6^{+8.1}_{-6.7}$ fb, respectively. The calculated production cross sections of a pair of S-wave charmonium states are shown in Fig. 2. Our new evaluation of the cross sections in the reaction $e^+ + e^- \to \mathcal{V}_{c\bar c} + \mathcal{P}_{c\bar c}$ evidently shows that the systematic account of all relativistic effects connected with the bound state wave functions, the gluon and quark propagators removes the discrepancy between theory and experiment. Numerically, the increase of the cross section σ (12) is determined approximately by the factor of 2 coming from the relativistic corrections entering in the production amplitude (2) (in this part our results agree with the previous calculations in Ref. [15]) and by another factor of 2 from the relativistic bound state wave functions. In our analysis we use the exact expressions (3)–(4) for the relativistic wave functions. Thus we correctly take into account all relativistic contributions of orders $O(v^2)$ and $O(v^4)$ since they are determined by the convergent momentum integrals due to the presence of the relativistic factors in the denominators of expressions (3)–(4). Therefore the resulting theoretical uncertainty is connected with the omitted terms of the employed truncated expansions (6), (7), (9) which are of order v^2p^2/s. Taking into account that the average value of the heavy quark velocity squared in the charmonium is $\langle v^2\rangle = 0.3$, we expect that they should not exceed 5–10% in the interval of energies $\sqrt{s} = 7$–11 GeV. We should remind also that our relativistic quark model has the phenomenological structure and differs significantly from the approach of nonrelativistic QCD (NRQCD). Despite the fact that it is based on the quantum field-theoretic approach, it contains a number of the phenomenological parameters which we fixed solving many tasks in the quarkonium physics. Unfortunately, we cannot control the theoretical accuracy in the same manner as in NRQCD. We obtained the theoretical predictions for the masses and decay rates of different charmonium states with more than one per cent accuracy. So, we suppose in this study that

[1] They are available from authors: apm@physik.hu-berlin.de.

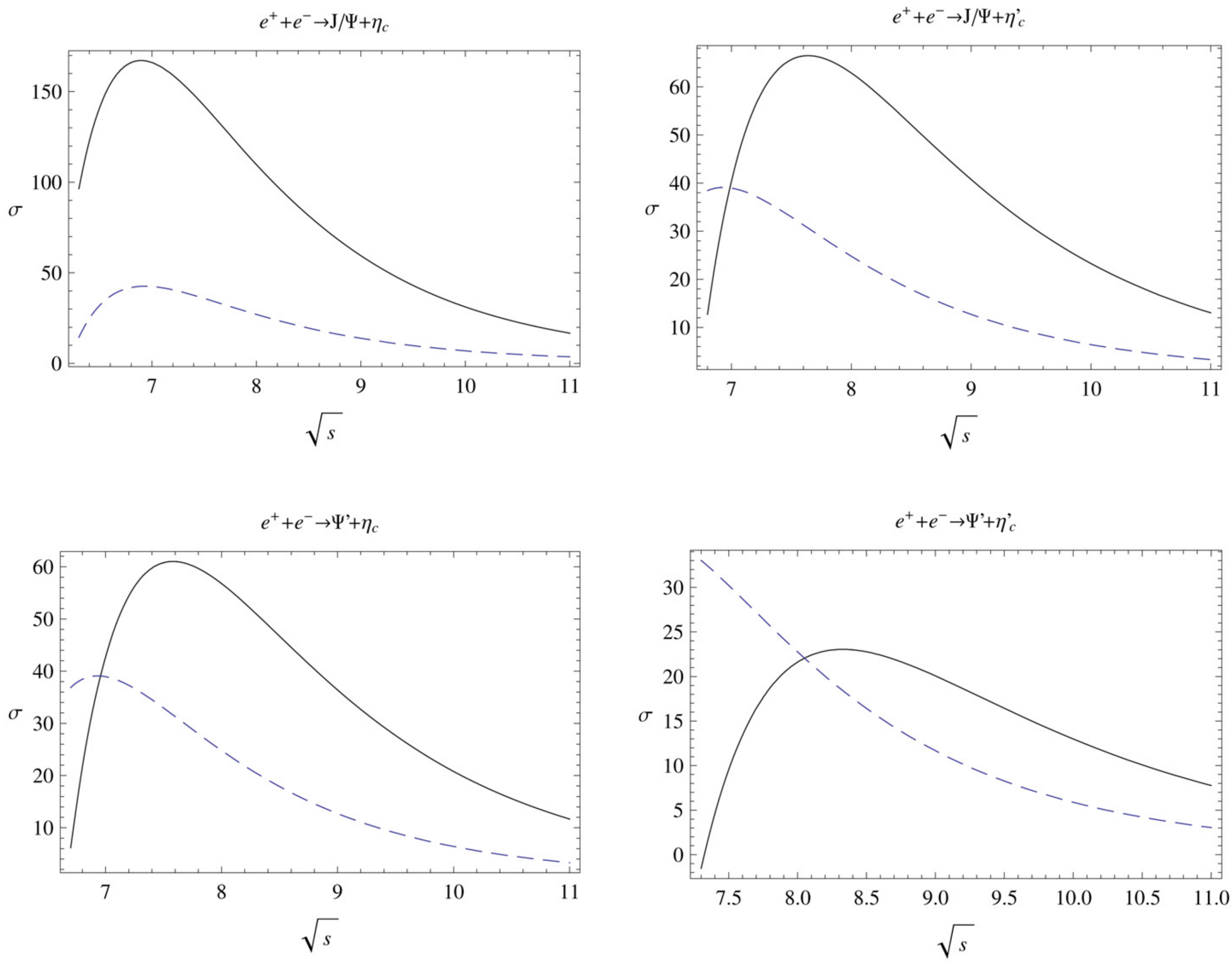

Fig. 2. The cross section in fb of e^+e^- annihilation into a pair of S-wave charmonium states with the opposite charge parity as a function of the center-of-mass energy $\sqrt{s}$ (solid line). The dashed line shows the nonrelativistic result without bound state and relativistic corrections.

Table 1
Comparison of the obtained results with previous theoretical predictions and experimental data.

State H_1H_2	$\sigma_{\text{BaBar}} \times \text{Br}_{H_2 \to \text{charged} \geqslant 2}$ (fb) [6]	$\sigma_{\text{Belle}} \times \text{Br}_{H_2 \to \text{charged} \geqslant 2}$ (fb) [5]	σ (fb) [12]	σ_{NRQCD} (fb) [7]	σ (fb) [9]	σ (fb) [7]	σ (fb) [15]	Our result (fb)
$\Psi(1S)\eta_c(1S)$	$17.6 \pm 2.8^{+1.5}_{-2.1}$	$25.6 \pm 2.8 \pm 3.4$	26.7	3.78	5.5	7.4	7.8	22.2 ± 4.2
$\Psi(2S)\eta_c(1S)$		$16.3 \pm 4.6 \pm 3.9$	16.3	1.57	3.7	6.1	6.7	15.3 ± 2.9
$\Psi(1S)\eta_c(2S)$	$16.4 \pm 3.7^{+2.4}_{-3.0}$	$16.5 \pm 3 \pm 2.4$	26.6	1.57	3.7	7.6	7.0	16.4 ± 3.1
$\Psi(2S)\eta_c(2S)$		$16.0 \pm 5.1 \pm 3.8$	14.5	0.65	2.5	5.3	5.4	9.6 ± 1.8

there are no additional essential theoretical uncertainties in the bound state wave functions connected with the formulation of our model in region of nonrelativistic momenta. In the region of relativistic momenta $p \geqslant m_c$ we have in the quasipotential approach a definite theoretical uncertainty in the determination of the bound state wave function. Direct evaluation of the momentum integrals entering in Eq. (12) shows that the region of relativistic momenta in which either $p \geqslant m_c$ or $q \geqslant m_c$ gives near 30% of the total value σ. This clearly demonstrates the importance of more exact determination of the bound state wave functions in the relativistic region. For comparison, the contribution of the relativistic region in which either $p \geqslant m_c/2$ or $q \geqslant m_c/2$ gives near 60% of the cross section (12). Assuming that the error in the determination of the wave functions $\psi_0^{\mathcal{P},\mathcal{V}}$ can amount 10% in the relativistic region $p, q \geqslant m_c$ (larger value of the error will lead to the essential discrepancy between the experiment and theory in the calculation of the charmonium mass spectrum) we obtain that the corresponding error in the cross section (12) is not exceeding 10%.

It is important to point out that it is not possible to simply compile the enhancements of the production cross sections originating from our calculation of the relativistic contributions and from the one-loop corrections calculated in Ref. [13]. The latter was done in the nonrelativistic limit. Indeed in our model the interaction potential in the relativistic wave equation contains the one-loop radiative corrections. Therefore the inclusion of the one-loop corrections considered in [13] in our calculation requires their complete recalculation using our relativistic wave functions, since we take into account effectively some part of the one-loop diagrams connected with the exchange of gluons between heavy quarks in the final state.[2] As a result both nonperturbative and partially perturbative contributions are taken into account.

Thus our approach cannot be directly confronted with the one of Ref. [13]. The radiative corrections are the main source of the theoretical uncertainty in our calculations. Indeed, available estimates of one-loop corrections in the nonrelativistic limit indicate that they are considerable. Taking their values from [13] (relativistic factor $K = 1.8$ to nonrelativistic result) we estimate that this part of the theoretical error should not exceed 15%. Therefore the total theoretical uncertainty amounts to 19% for the energy region $\sqrt{s} = 10.6$ GeV. To obtain this estimate we add the above mentioned relativistic and one-loop uncertainties in quadrature (as it was done in Ref. [19]). These theoretical errors in the calculated production cross section at $\sqrt{s} = 10.6$ GeV are shown directly in Table 1. There are no additional uncertainties related to the choice of m_c or any other parameters of the model, since their values were fixed from our previous consideration of meson and baryon properties [24–27,29].

In summary, we presented a systematic treatment of relativistic effects in the double charmonium production in e^+e^- annihilation. We explicitly separated two different types of relativistic contributions to the production amplitudes. The first type includes the relativistic v/c corrections to the wave functions and their relativistic transformations which were for the first time exactly taken into account. The second type includes the relativistic $p/\sqrt{s}$ corrections emerging from the expansion of the quark and gluon propagators. The latter corrections were taken into account up to the second order. It is important to note that the expansion parameter $p/\sqrt{s}$ is very small. Contrary to the previous calculations within NRQCD all obtained expressions for the relativistic contributions are now expressed through converging integrals. Thus no additional uncertainty related to their regularization emerges. Therefore we can reliably estimate the uncertainty originating from the neglected higher-order relativistic contributions. The calculated values for the production cross sections agree well with experimental data.

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Appendix A. The coefficients ω_{ij}, σ_{ij}, ρ_{ij}, γ_i entering in the production cross section

$$\omega_{00} = -18(-1+2\kappa-3u)(u-1)^2u^2, \tag{A.1}$$

$$\omega_{01} = 6(u-1)^2\big[32\kappa^3+16\kappa^2(-5+u)-6\kappa u^2+3(1-5u)u^2\big], \tag{A.2}$$

$$\omega_{10} = 6u^2\big[96\kappa^3-34\kappa(u-1)^2+(u-1)^2(5u-1)-16\kappa^2(1+11u)\big], \tag{A.3}$$

$$\begin{aligned}\omega_{11} = {}&-2\big[536\kappa^5+102\kappa(u-1)^2u^2+3(u-1)^2u^2(1+3u)-8\kappa^4(61+67u)\\&-6\kappa^3\big(114+u(161u-228)\big)-2\kappa^2\big(46+u\big(-170+u(175+237u)\big)\big)\big],\end{aligned} \tag{A.4}$$

$$\omega_{12} = -96\kappa u^2\big[6\kappa^2-2(u-1)^2-\kappa(1+11u)\big], \tag{A.5}$$

$$\omega_{21} = -96\kappa^2(u-1)^2(-5+2\kappa+u), \tag{A.6}$$

$$\begin{aligned}\gamma_1 = {}&64\kappa^3\big[64\kappa^6\big(5+2u(2u-5)\big)+32\kappa^5\big(4+(u-1)u(14+3u)\big)+4\kappa^2(u-1)^2\big(146+u\big(-584+u\big(851+6u(21u-89)\big)\big)\big)\\&-16\kappa^4\big(44+u\big(u\big(273+u(49u-194)\big)-176\big)\big)-16\kappa^3\big(7+u\big(u\big(37+u\big(u-25+15u^2\big)\big)-27\big)\big)\\&-(u-1)^2\big(194+u\big(-1164+u\big(2754+u\big(-3256+3u\big(676+27(u-8)u\big)\big)\big)\big)\big)\\&+2\kappa(u-1)\big(2+u\big(-108+u\big(523+u\big(-992+u\big(874+3u(9u-106)\big)\big)\big)\big)\big)\big],\end{aligned} \tag{A.7}$$

$$\begin{aligned}\gamma_2 = {}&-128\kappa^4(1-2u)^2\big[-31+32\kappa^5+2\kappa(1-3u)^2(1+u)^2-16\kappa^4(1+3u)-16\kappa^3\big(1+u(5u-2)\big)\\&+8\kappa^2\big(7+u\big(-17+u(11+15u)\big)\big)+u\big(151-u\big(286+3u\big(-98+u(55+9u)\big)\big)\big)\big],\end{aligned} \tag{A.8}$$

$$\begin{aligned}\sigma_{01} = {}&96\kappa^2(u-1)^2\big[-17+2\kappa(1-2u)^2+16\kappa^4(-1+u)-8\kappa^2(u-1)\big(4+u(5u-8)\big)\\&+u\big(81+u\big(-144+u\big(124-57u+9u^2\big)\big)\big)\big],\end{aligned} \tag{A.9}$$

$$\begin{aligned}\sigma_{10} = {}&48\kappa^2u^2\big[-1+32\kappa^5+16\kappa^4(1-5u)-16\kappa^3\big(1+u(5u-2)\big)+8\kappa^2(5u-1)\big(1+u(5u-2)\big)\\&+6\kappa\big(1+u(3u-2)\big(2+u(2+u)\big)\big)-u\big(7+u\big(-34+u\big(2+51u+45u^2\big)\big)\big)\big],\end{aligned} \tag{A.10}$$

$$\begin{aligned}\sigma_{11} = {}&\kappa^2\big[600-8\kappa\big(-1695+2\kappa\big(-45+\kappa\big(1527+2\kappa\big(11+\kappa\big(-659+2\kappa(67\kappa-31)\big)\big)\big)\big)\big)-4328u\\&+16\kappa\big(-5085+\kappa\big(-769+2\kappa\big(3054+\kappa\big(-351+2\kappa(97\kappa-659)\big)\big)\big)\big)u\\&+4\big(3235+\kappa\big(49319+4\kappa\big(2783+\kappa\big(-9443+\kappa(1071+1681\kappa)\big)\big)\big)\big)u^2\\&-4\big(4987+4\kappa\big(15419+\kappa\big(3801+667\kappa(\kappa-10)\big)\big)\big)u^3+2\big(7933+\kappa\big(83683+2(7829-8265\kappa)\kappa\big)\big)u^4\\&-2\big(3711+29142\kappa+3770\kappa^2\big)u^5+(4765+9851\kappa)u^6+983u^7\big],\end{aligned} \tag{A.11}$$

[2] This is beyond the scope of the present Letter.

$$\rho_{01} = 96\kappa^2(u-1)^2\big[16 + 12\kappa^2(u-1) + u\big(-40 + (33-13u)u\big) - 2\kappa\big(2 + (u-4)u\big)\big], \tag{A.12}$$

$$\rho_{10} = 24\kappa u^2\big[48\kappa^4 + 16\kappa^3(1-7u) + \kappa^2\big(-4 + 8u - 76u^2\big) - (u-1)^2\big(-7 + u(14+3u)\big) + 2\kappa\big(-1 + u\big(-1 + u + 73u^2\big)\big)\big], \tag{A.13}$$

$$\begin{aligned}\rho_{11} = {} & -8\kappa\big[528\kappa^6 - 128\kappa^5(3+5u) + 3(u-1)^2u^2\big(-7 + u(14+3u)\big) - 12\kappa^4\big(67 + 2u(56u-67)\big) \\ & + 4\kappa^3\big(97 + u\big(-259 + 6u(39+9u)\big)\big) + \kappa^2\big(814 + u\big(-3256 + u\big(5017 + u(-3522 + 1159u)\big)\big)\big) \\ & + \kappa\big(70 + u\big(-382 + u\big(969 + u\big(-1323 + 2u(469+80)\big)\big)\big)\big)\big],\end{aligned} \tag{A.14}$$

$$\rho_{21} = -\rho_{01}, \qquad \rho_{12} = -\rho_{10}, \qquad \sigma_{21} = -\sigma_{01}, \qquad \sigma_{12} = -\sigma_{10}.$$

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